

Instructions:

1. Write your *Name*, *PID*, *Section Number*, and *Exam Version* on the *front* of your blue book.
2. Draw the following table on the inside cover of your blue book:

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3. You may use one 8.5x11 in. sheet of *handwritten* notes, but no books or other assistance during this exam.
4. No calculator, phones, or any other electronic devices are allowed during this exam.
5. Present your solutions clearly in your Blue Book:
 - (a) Carefully indicate the number and letter of each question and each part of a question.
 - (b) Present your answers in the same order as they appear in the exam.
 - (c) Start each problem on a new page.
6. Show all of your work. Unsupported answers will receive no credit.
7. Turn in your exam paper and your note sheet with your Blue Book.

0. (1 point.) Carefully read and follow the instructions.
1. (6 points.) Find all critical points of the function

$$f(x, y) = 2y^3 - 6xy + 3x^2$$

and classify each of them as a local maximum, a local minimum, or a saddle point.

2. (a) (4 points.) Calculate the tangent plane to the graph of the function

$$f(x, y) = xe^{x+y^2}$$

at the point $(-1, 1)$.

- (b) (2 points.) Use linear approximation to estimate $f(-1.1, 0.8)$.

3. (a) (4 points.) A farmer from Omaha, Nebraska is tending to his corn field in the dead of winter. Unfortunately he forgot his mittens and is *freezing*. The temperature in the field is given by the function $T(x, y) = \frac{1}{2x^2 + 1y^2}$ where x is measured in latitude and y is measured in longitude. If the farmer is standing at the point $(a, b) = (-1, -2)$, in which direction should he travel to warm up the fastest? Your answer should be a vector in 2 dimensions. *Note: this function is not actually a realistic depiction of Temperature!*
- (b) (2 points.) Sadly, this farmer never took Calc III and has no idea what to do. He ends up walking in a straight line towards the point $(-3, -1)$. Is the farmer warming up or getting colder? How do you know?
4. (6 points.) Suppose $f(x, y) = x^2 - xy$ and $x(s, t) = \frac{t}{s}$, $y(s, t) = s^2 + t$. Using the *multivariable chain rule* calculate the partial derivatives

$$\frac{\partial f}{\partial s} \Big|_{(s,t)=(2,1)} \quad \frac{\partial f}{\partial t} \Big|_{(s,t)=(2,1)}$$

WARNING: answers that do not use the chain rule will receive no credit.

5. (extra credit: 1 pt.) Is it possible for a *differentiable* function $g(x, y)$ to have the following partial derivatives?

$$g_x(x, y) = xe^y \cos(x) \quad g_y(x, y) = ye^x \cos(y)$$

Explain.